Small Area Estimation for Government Surveys

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Abstract: In the past three years, we developed decision-based estimation to estimate survey totals for the Annual Survey of Public Employment and Payroll (ASPEP). Recently, we developed a step-wise ratio method to estimate the Annual Finance Survey (AFS) characteristics of interest: Revenues, Expenditures, Debts, and Assets. In this paper, we discuss some small area challenges when we estimate survey totals at a function level, where the small area issue occurs. First, we introduce the idea of using synthetic estimation and modified direct estimation in ASPEP. Then, we modify the composite estimator as a weighted average of the modified direct estimator and synthetic estimator. We also apply the empirical Bayes to estimate the survey total, and then compare it to the modified composite using 2007 Census of Governments data. Secondly, we introduce the idea of using a step-wise ratio in the AFS with a validation the from New York census data.

Key Words: Decision-based Estimation, Modified Direct Estimator, Synthetic Estimation, Composite Estimation, Step-Wise Ratio

I. Introduction

In survey analysis it is common to have a problem of small sample size, or an area where there were no sampled units. A direct estimator like Horvitz-Thompson estimator does not work well in very detailed categories areas with a small sample size or no sampled units. Small area issues can appear from other aspects like the ones in our government surveys: Annual Survey of Public Employment and Payroll (ASPEP), and Annual Finance Survey (AFS). Those two surveys have the same sample design. The sampled units were stratified by state and type (see below). However, in publication it was required that the reported estimates are estimated by state and its functionality (see below). This leads to Small Area Estimation (SAE) challenges. We describe briefly ASPEP and AFS characteristics below.

The Annual Survey of Public Employment and Payroll (ASPEP) produces statistics on the number of federal, state, and local government employees and their gross payrolls. For more information on the survey, please see the Website for ASPEP http://www.census.gov/govs/apes/. ASPEP provides current estimates for full-time and part-time state and local government employment and payroll by government function (i.e., elementary and secondary education, higher education, police protection, fire protection, financial administration, judicial and legal, etc.). ASPEP covers all states and

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local governments in the United States, which include counties, cities, townships, special districts, and school districts. The first three types of government are referred to as general-purpose governments, because they generally provide multiple government activities. Activities are coded as function codes. School districts cover only education functions. Special districts usually provide only one function, but can provide two or three functions. ASPEP is the only source of public employment data by program function and selected job category. Data on employment include number of full-time and part-time employees, gross pay, and hours paid for part-time employees. Reported data are for the government's pay period that includes March 12. Data collection begins in March and continues for about seven months.

There are 89,526 state and local government units in our universe. In 2009, after exploring possible cut-off sample methods for ASPEP, we developed a new modified cut-off sample method based on the current systematic stratified probability proportional-to-size (PPS) sample design. This method reduced the sample size, which saved resources, improved the precision of the estimates, reduced respondent burden, and improved data quality. The modified cut-off sample method was applied in two stages. We first selected a state-by-governmental type stratified PPS sample. The PPS sample was based on total payroll, which was the sum of full-time pay and part-time pay, from the Employment portion of the 2007 Census of Government. In the second stage, we constructed a cut-off point to distinguish small and large government units in the stratum. Lastly, we sub-sampled the strata with small-size government units with a simple random sampling method.

ASPEP was designed to estimate survey totals of key variables: full-time employment, full-time payroll, part-time employment, part-time payroll, part-time hours, full-time equivalent employment, total payroll, and total employment. Cheng et al. (2009) proposed a method, Decision-based, to improve the precision of estimates and reduce the mean square error of weighted survey total estimates. Basically, the Decision-based method combined the strata to improve the models by testing the equality of the slopes of regression models from different strata. In Cheng et al. (2009), the hypothesis test was carried out in two steps. First, a test was performed of the null hypothesis that the slopes were identical. If the p-value was less than 0.05, the null hypothesis would be rejected to conclude that the regression lines were significantly different. In this case, there was no reason to compare the intercepts. If the p-value was greater than 0.05, the null hypothesis of equality of slopes could not be rejected, but intercepts could be compared. If the regression lines for the two substrata were not found to be significantly different, then a single line was estimated from the combined substrata. The Decision-based estimates provided a fundamental base to improve the reliability of the indirect small area estimation.

As mentioned earlier ASPEP's sampled units were stratified by state and government types. However, it was required to estimate the variables of interest at the state and functional code level, which contained up to 30 categories for each government unit. This naturally brought the small area challenges, because we did not have any control on the sample size at the state and function code level. For example, the sample size for the state of Maryland was 48. But, there were only 3 sample units with the airport activity, labeled as function code of 001. In the worst case, we have no sample for some specific function codes. If there were missing data in some specific function for a government unit, these missing data could be structural zeros. We define that structural zeros to be cells in which observations are impossible. Table 1 shows that each government unit in a state may have different functions. Table 2 lists all government function codes.

Table 1: There are structural zeros in the government unit

	GOVERNMENT UNITS							
FUNCTION	**1**	**2**	**3**	**4**	**5**	**…**	**N-1**	**N**
Airport	✓	X	X	X	X	…	✓	X
Correction	✓	✓	X	✓	✓	…	✓	✓
Elementary/Second	✓	✓	✓	✓	X	…	X	✓
Financial	X	✓	✓	✓	✓	…	✓	✓
FireFighters	✓	✓	✓	✓	✓	…	✓	✓
…	…	…	…	…	…	…	…	…
Fire	✓	✓	✓	✓	✓	…	✓	✓
Police	✓	X	✓	✓	✓	…	✓	✓

In contrast, AFS's variables are in dollars amounts and have a different set of function codes. Please refer to the following link to have more detail in the function code definition:
http://www.census.gov/govs/www/06classificationmanual/06_gfe_classmanual_toc.html

The finance function codes are categorized into four sectors: Revenue, Expenditure, Debts, and Assets. Each sector contains a set of function codes defined in the classification manual. First, we estimate the dollar amounts for each combination of state and function code. Secondly, we aggregate them into each sector.

Both ASPEP and AFS have the same design. They have the same SAE challenge. However, ASPEP has a relatively good linear relationship with previous data but AFS does not. Therefore, the SAE application in each survey has different forms. Basically, SAE methods borrow strength from related or similar small areas using auxiliary data. In ASPEP, it is in the form of synthetic. In AFS it is in the form of step-wise ratio. We describe the idea of SAE and each estimation method in subsequent sections.

Table 2: Function codes in the Annual Survey of Public Employment and Payroll

ItemCode	Meaning
000	Totals for Government
001	Airports
002	Space Research & Technology (Federal)
005	Correction
006	National Defense and International Relations (Federal)
012	Elementary and Secondary - Instruction
112	Elementary and Secondary - Other Total
014	Postal Service (Federal)
016	Higher Education - Other
018	Higher Education - Instructional
021	Other Education (state)
022	Social Insurance Administration (state)
023	Financial Administration
024	Firefighters
124	Fire - Other
025	Judicial and Legal
029	Other Government Administration
032	Health
040	Hospitals
044	Streets & Highways
050	Housing & Community Development (Local)
052	Local Libraries
059	Natural Resources
061	Parks & Recreations
062	Police Protection - Officers
162	Police - Other
079	Welfare
080	Sewerage
081	Solid Waste Management
087	Water Transport & Terminals
089	Other & Unallocable
090	Liquor Stores (state)
091	Water Supply
092	Electric Power
093	Gas Supply
094	Transit

II. Methodology

What is Small Area? Traditionally, Small Area is a small geographic area within a larger geographic area or a small demographic group within a larger demographic group. The sample size in the domain of interest is too small to use a standard estimator. For surveys of governments, small area refers to their state by function. Most small area estimation methods borrow strength from related or similar areas using auxiliary data. There is growing demand from the public for reliable small area statistics. At the design stage, we can't consider attaining precision at the state and function code level because it would force the sample to be too large. However, we have to handle this challenge at the estimation stage.

II.A 2009 Annual Survey of Public Employment and Payroll

Let g represent the state and f represent the function code. We want to estimate the total of employees or payroll information at the state by function level:

$$Y_{gf} = \sum_{i \in U_{gf}} Y_{gfi}$$

where U is the universe of function codes in all states, and U_{gf} is the universe of function code f, state g. Thus, U_{gf} is subset of U, that is, $U_{gf} \subset U$. The sample size for function code f, n_f, is less than or equal to the sample size n, that is, $n_f \leq n$. The domain of sample for function code f of state g is the intersection of the sample domain of state g and the universe of function code f and state g, $S_{gf} = S_g \cap U_{gf}$.

In some cases, the changes in Employment statistics are relatively stable. Therefore, a linear regression is suitable for some state by government type cells as done prior to Fiscal Year (FY) 2009. However, due to small sample sizes and poor fit for many cells, a small area estimation method (SAE) is more appropriate. SAE is only applied for the PPS sample. For certainties, the direct estimate was used. Information on Births units is not available at the sampling stage. Therefore, we sample Births separately from the PPS and Certainties sample.

Figure 1 briefly shows how we estimated the variable of interest in each cell of state by function code table. We applied the design-based direct estimator (Horvitz-Thompson), and the synthetic estimator in each cell. The direct estimator has high variability due to the small sizes. On the other hand the synthetic estimator reduces the variability but introduces some bias. Therefore, we introduce the composite estimator, which is a weighted average of those two estimators. We also modified the direct estimator (modified direct) by borrowing strength from similar cells to smooth the direct estimator. We will go through each of our estimators in detail in subsequent sections.

In this section, we discuss how to estimate Y_{gf} for a given state g and function code f. Here, Y_{gf} represents the survey total of key variables: full-time employment, full-time payroll, part-time employment, part-time payroll, part-time hours, full-time equivalent employment, total payroll, and total employment. We describe all the estimators used in our estimation process: Direct (Horvitz-Thompson), Decision-based, Synthetic, Composite, Modified Direct, and the Composite estimator.

Figure 1: Cross-Tabulation of State by Function

Function Code (f)		State (g)				
		1	2	g	...	51
	001 Airports					
	005 Correction					
	...					
	f			$\hat{y}_{gf}$		
	...					
	162 Police					
		$\hat{Y}_1$	$\hat{Y}_2$	$\hat{Y}_g$		$\hat{Y}_{51}$

2009 ASPEP regress on 2007 Census (decision-based)

II.A.1 Direct estimator (Horvitz-Thompson)

A general design-based direct estimator for the total is:

$$\hat{t}_{y,gf} = \sum_{i \in S} w_{gfi} y_{gfi}. \qquad (1)$$

where the weight, $w_{gfi} = \dfrac{1}{\pi_{gfi}}$, and $\pi_{i,gf}$ is the inclusion probability for unit i in state g and function code f. In this paper, we also denote $\hat{t}_{y,gf}$ as $\hat{Y}_{gf}^{HT}$.

II.A.2 Decision-based estimator

The Decision-based (DB) method helps to estimate the synthetic in each cell by providing a stable state total as a reliable estimator in a large area covering all small areas, states by function code level. In other words it was used for estimating the aggregates. DB was a process of testing the possibility of combining the strata in other to get a better estimate of the total. This method strengthened the statistical models for the area of estimation. The state total was estimated by a single stratum weighted regression (GREG) estimator specified as follows:

$$\hat{t}_{y,GREG} = \hat{t}_{y,\pi} + \hat{b}(t_x - \hat{t}_{x,\pi}) \qquad (2)$$

where $t_x = \sum_{i \in U} x_i$, $\hat{t}_{x,\pi} = \sum_{i \in S} \dfrac{x_i}{\pi_i}$, $\hat{t}_{y,\pi} = \sum_{i \in S} \dfrac{y_i}{\pi_i}$, $\hat{b} = \dfrac{\sum_{i \in S} (x_i - \bar{x})(y_i - \bar{y})/\pi_i}{\sum_{i \in S} (x_i - \bar{x})^2/\pi_i}$, where π_i is the inclusion probability, and x_i is the auxiliary data from the Employment portion of the Census of Governments for government unit *i*.

The slope $\hat{b}$ was obtained by the Decision-based (DB) process proposed by Cheng et al. (2009). The DB method improved the precision of estimates and reduced the mean square error of weighted survey total estimates. The idea was to test the equality of linear regression lines to determine whether we can combine data in different substrata. The null hypothesis $H_0 : b_1 = b_2$, that is, the equality of the frame population regression slopes for two substrata. In large samples, $\hat{b}$ is approximately normally distributed, $\hat{b} \sim N(b, \Sigma)$. Under the null hypothesis, with two sub-strata U_1, U_2 (large and small) from samples S_1, S_2 of sizes n_1, and n_2, we have $\hat{b}_1 - \hat{b}_2 \sim N(0, \Sigma_{1,2})$ where $\hat{b}_1 \sim N(b, \Sigma_1)$, $\hat{b}_2 \sim N(b, \Sigma_2)$, and $\Sigma_{1,2} = \Sigma_1 + \Sigma_2$. Therefore, the test statistic is

$$(\hat{b}_1 - \hat{b}_2)\Sigma_{1,2}^{-1}(\hat{b}_1 - \hat{b}_2) \sim \chi_1^2 \tag{3}$$

Our research showed that it was unnecessary to test the hypothesis for the intercept equality because our data analysis showed that we never rejected the null hypothesis of equality of intercepts when we could not reject the null hypothesis of equality of slopes. This is reasonable because the 2007 payroll could be 0 essentially only if the 2002 payroll was.

We will discuss the variance estimator for $\hat{b}$ in Section 3. The critical value for a test based on (3) was obtained from a chi-squared distribution with 1 degree of freedom. The test was performed with a significance level of $\alpha = 0.05$. If we could not reject the null hypothesis, then the slopes estimated in sub-strata S_1 and S_2 were accepted as the same, and the Decision-based estimator was equal to the GREG estimator for the union of two sample sets, that is, for $S = S_1 \cup S_2$. Otherwise, the Decision-based estimator would be the sum of two separate GREG estimators of stratum totals, that is,

$$\hat{t}_{y,DB} = \begin{cases} \hat{t}_{y,greg} & \text{if } \mathrm{H}_0 \text{ is accepted} \\ \sum_{h=1}^{2} \hat{t}_{y,greg}^{h} & \text{if } \mathrm{H}_0 \text{ is rejected.} \end{cases} \tag{4}$$

where $\hat{t}_{y,greg}$ denotes the GREG estimator from the combined stratum S, while $\hat{t}_{y,greg}^{h}$ denotes the GREG estimator from substratum h from sample S_h. DB produced 51 (50 states and Washington D.C.) totals for each key variable.

II.A.3 Synthetic estimation

Synthetic estimation assumes that small areas have the same characteristics as large areas, and there is a reliable estimate for large areas. There are many advantages of synthetic estimation. They are accurate, simple and intuitive, aggregated estimates, that can be applied to all sample designs, and borrow strength from similar small areas.

Synthetic estimation can even provide estimates for areas with no sample from the sample survey, and it does not need a study model.

The general idea for synthetic estimation is that if we have a reliable estimate for a large area and this large area covers many small areas, then we can use this estimate to produce an estimate for a small area. The key element for calculating the synthetic estimation for a small area (state by function code level) is to estimate the proportion of that small area of interest within the large state area. This estimate for the small area is known as the synthetic estimate.

The synthetic estimator for function code *f* of state *g* is:

$$\hat{Y}_{gf}^{S} = \frac{x_{gf}}{\sum_{f} x_{gf}} \hat{t}_{g}^{DB} \tag{5}$$

where x_{gf} is the auxiliary information which is obtained from the Employment portion of Census of Government and the state total, $\hat{t}_{g}^{DB}$ is obtained by the Decision-based estimate from equation (4).

II.A.4 Composite estimator

To balance the potential bias of the synthetic estimator, $\hat{Y}_{gf}^{S}$, against the instability of the design-based direct estimator, $\hat{Y}_{gf}^{HT}$, we introduce a composite estimator as a weighted average of these two estimators. Thus, the composite estimate was applied on the PPS sample for each state by function code cell. Generally, it has the form:

$$\hat{Y}_{gf}^{C} = \phi_g \hat{Y}_{gf}^{HT} + (1-\phi_g)\hat{Y}_{gf}^{S}. \tag{6}$$

where $\hat{\phi}_g = 1 - \frac{\sum_f \mathrm{var}(\hat{y}_{gf}^{HT})}{\sum_f (\hat{y}_{gf}^{S} - \hat{y}_{gf}^{HT})^2}$ (Purcell & Kish, 1979). In some cases, we observed negative $\hat{\phi}$. To fix this problem, we applied the method which was introduced by Lahiri and Pramanik (2010).

II.A.5 Modified direct estimator

We replaced the direct $\hat{Y}_{gf}^{HT}$ in (6) by a modified direct estimate (MD), $\hat{Y}_{gf}^{MD}$, due to instability of the design-based direct estimate caused by small sizes. The modified direct estimator from Rao's Small Area Estimation (2003) is given as:

$$\hat{Y}_{gf}^{MD} = \hat{Y}_{gf\pi}^{HT} + \hat{b}_f (X_{gf} - \hat{X}_{gf\pi}^{HT}) \tag{7}$$

where

$$\hat{Y}^{HT}_{gf,\pi} = \sum_{i \in S_{gf}} \frac{y_{gfi}}{\pi_{gi}},\ X_{gf} = \sum_{i \in U_{gf}} x_{gfi},\ \hat{X}^{HT}_{gf,\pi} = \sum_{i \in Sgf} \frac{x_{gfi}}{\pi_{gi}}, \text{ and}$$

$$\hat{b}_f = \frac{\sum_{g \in G, i \in S_{gf}} (x_{gfi} - \overline{x}_f)(y_{gfi} - \hat{y}_f) / \pi_{gi}}{\sum_{g \in G, i \in S_{gf}} (x_{gfi} - \overline{x}_f)^2 / \pi_{gi}}$$

Since the modified direct estimators use data from outside the domain, we can see that the MD method is smoothed by borrowing strength across the state. The estimator $\hat{Y}^{MD}_{gf}$ is approximately unbiased as the overall sample size increases, even if the domain sample size is still small. The modified direct estimator (7) is performed under some conditions which allowed producing a reliable $\hat{b}_f$, for example, goodness of fit R^2, slopes, and the sample sizes.

II.A.6 Modified composite estimator

With MD estimator available, we can modify the composite estimator as:

$$\hat{y}^{C}_{gf} = \phi_g \hat{y}^{MD}_{gf} + \left(1 - \phi_g\right) \hat{y}^{S}_{gf} \tag{8}$$

We can re-write the MD estimator as:

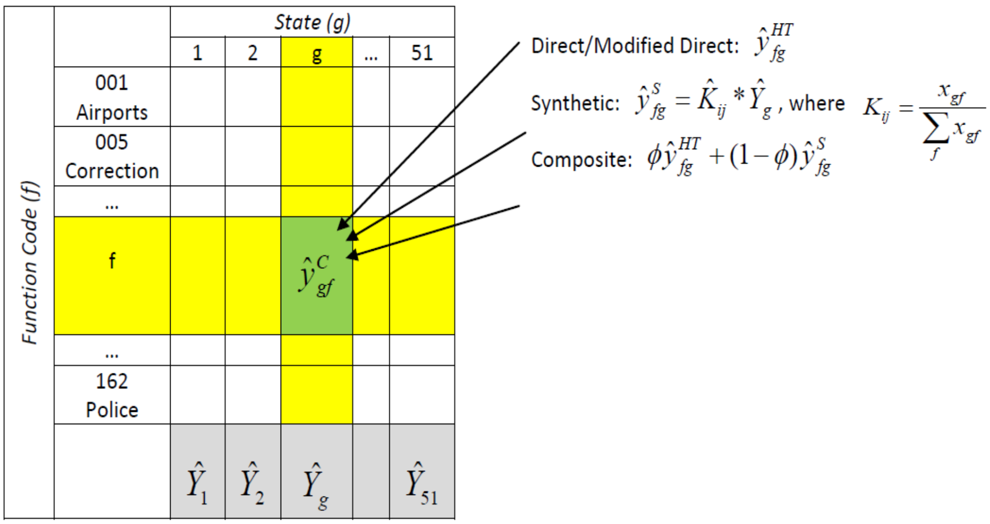
$$\hat{Y}^{MD}_{gf} = X_{gf} * \hat{b}_f + \sum_{j \in S_{gf}} w_j e_j \tag{9}$$

where

$$e_j = y_j - X_j * \hat{b}_f$$

The first term $X_{gf}\hat{b}_f$ is the synthetic regression estimator and the second term, $\sum_{j \in S_{gf}} w_j e_j$ approximately corrects the bias of the synthetic estimator. Figure 2 shows all the estimators we discuss in this paper.

Figure 2: Cross-Tabulation of State by Function
Estimators in Each Cell



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II.A.7 Variance Estimation

The variance estimate is required for each cell identified by state and function code level. Due to the complexity of the two-stage sampling design with the cut-off technique, we calculated the approximate variance (AV) of the composite estimator. AV is estimated on the non sub-sampling sample. Besides, there are B&N units, which are very small and contribute a small amount in the survey total. We assume the variance on B&N is ignorable.

The coefficient of variance, CV, is estimated by $\frac{\sqrt{Var(\hat{y})}}{\hat{y}}$, where $\hat{y}$ is the composite estimate from the PPS units, certainties, and B&N.

We applied a Taylor series method to estimate the approximate variance for the estimates derived in the previous section for each cell. For composite estimation, we estimate the mean square errors instead of approximate variance because of the bias from the synthetic estimation. We calculate the variance of the direct (Horvitz-Thompson) estimates and mean square error of the synthetic estimates, and then the mean square errors of the composite estimation is as follows:

$$\widehat{MSE}(\hat{y}^{C}) = \phi^2 \widehat{var}\,(\hat{y}^{HT}) + (1-\phi)^2 \widehat{MSE}(\hat{y}^{S})$$

For simplicity, we assumed there was no correlation between the design-based direct estimate and the synthetic estimate.

Note: DC and Hawaii had CV = 0 because they are censuses.

II.B 2009 Annual Finance Survey

AFS provides statistics on four categories: Revenue, Expenditure, Debt, and Asset (cash and security holdings). There are statistics for the 50 states areas and the District of Columbia, as well as a national summary. Statistics also are available by level of government: state, local, and state plus local aggregate. As in ASPEP, AFS uses the modified cut-off sampling method on stratified PPS units (see **I.**). The size is the maximum of dollar amounts among the four categories. Like ASPEP, sampled units in AFS are stratified by state and type but the reported aggregates are for state by function codes. For more information on the survey, please see the Website for ASPEP http://www.census.gov/govs/classification/index.html. Figure 3 is a snapshot for 2009- State and Local Finances Revenue

Figure 3: 2009 State and Local Finances (Revenue)

State and Local Government Finances by Level of Government: 2009

(Dollars in thousands. Coefficients of variation in percentages)

Description	United States				
	State and local government amount[1]	State and local government coefficient of variation	State government amount	Local government amount[1]	Local government coefficient of variation
Revenue[1]	**2,066,665,708**	**0.09**	**1,123,226,058**	**1,433,817,792**	**0.14**
General revenue[1]	2,413,384,189	0.07	1,495,730,319	1,408,032,013	0.13
Intergovernmental revenue[1]	536,760,320	0.10	495,623,675	531,514,788	0.16
From federal government	536,760,320	0.10	475,952,532	60,807,788	0.89
From state government[1]	([1])	(X)	–	470,706,999	0.14
From local governments[1]	([1])	(X)	19,671,143	([1])	(X)
General revenue from own sources	1,876,623,869	0.09	1,000,106,644	876,517,225	0.19
Taxes	1,271,355,992	0.10	715,496,219	555,859,773	0.24
Property	424,014,170	0.28	12,964,188	411,049,982	0.29
Sales and gross receipts	433,556,015	0.12	344,567,991	88,988,024	0.57
General sales	291,045,219	0.16	228,728,864	62,316,355	0.73
Selective sales	142,510,796	0.16	115,839,127	26,671,669	0.87
Motor fuel	37,815,730	0.11	36,471,286	1,344,444	3.02
Alcoholic beverage	5,865,179	0.45	5,347,705	517,474	5.08
Tobacco products	17,157,014	0.02	16,689,547	467,467	0.55
Public utilities	28,376,118	0.61	14,898,003	13,478,115	1.29
Other selective sales	53,296,755	0.27	42,432,586	10,864,169	1.32
Individual income	270,517,726	0.06	245,880,786	24,636,940	0.64
Corporate income	45,979,954	0.02	39,277,558	6,702,396	0.13
Motor vehicle license	21,296,295	0.12	19,626,624	1,669,671	1.51
Other taxes	75,991,833	0.30	53,179,072	22,812,761	0.99
Charges and miscellaneous general revenue	605,267,877	0.16	284,610,425	320,657,452	0.31
Current charges	388,766,090	0.24	161,238,746	227,527,344	0.41
Education	115,641,628	0.03	89,846,450	25,795,178	0.12
Institutions of higher education	99,739,656	0.02	88,603,712	11,135,944	0.21
School lunch sales (gross)	6,963,350	0.14	35,283	6,928,067	0.14
Hospitals	103,974,544	0.40	39,235,615	64,738,929	0.64
Highways	11,842,647	0.50	6,770,119	5,072,528	1.18
Air transportation (airports)	18,073,666	0.39	1,276,602	16,797,064	0.42
Parking facilities	2,023,611	2.36	13,987	2,009,624	2.37
Sea and inland port facilities	3,921,136	1.86	1,079,691	2,841,445	2.57
Natural resources	3,985,839	5.24	2,496,535	1,489,304	14.03
Parks and recreation	9,016,744	1.54	1,516,742	7,500,002	1.85
Housing and community development	5,852,741	1.86	799,389	5,053,352	2.15
Sewerage	39,453,377	1.55	506,688	38,946,689	1.57
Solid waste management	15,015,161	1.40	486,068	14,529,093	1.44
Other charges	59,964,997	0.71	17,210,860	42,754,137	0.99

We used the step-wise ratio method in the 2009 AFS estimation because the AFS does not have a good linear relationship with previous data; moreover some AFS data are quite volatile across years. We used three year data (2007, 2008, and 2009) to construct step-wise ratios. The step-wise ratios were constructed as follows:

$$\hat{R}_1 = \begin{cases} \dfrac{\sum_{i \in (S_{08} \setminus C) \cap U_{07}} w_{08} y_{08}}{\sum_{i \in (S_{08} \setminus C) \cap U_{07}} w_{08} y_{07}} = \hat{R}_{0708} \\ \dfrac{\sum_{i \in S_{08} \cap U_{07}} w_{08} y_{08}}{\sum_{i \in S_{08} \cap U_{07}} w_{08} y_{07}} = \hat{R}_{0708C} \\ \dfrac{\sum_{i \in S_{08}} w_{08} y_{08} / \sum_{i \in S_{08}} w_{08}}{\sum_{i \in U_{07}} y_{07} / N_{07}} = AVG\_\hat{R}_{0708} \end{cases}$$

$$\hat{R}_2 = \begin{cases} \dfrac{\sum_{i \in S_{08} \cap (S_{09} \setminus C)} w_{09} y_{09}}{\sum_{i \in S_{08} \cap (S_{09} \setminus C)} w_{09} y_{08}} = \hat{R}_{0809} \\ \dfrac{\sum_{i \in S_{08} \cap S_{09}} w_{09} y_{09}}{\sum_{i \in S_{08} \cap S_{09}} w_{09} y_{08}} = \hat{R}_{0809C} \\ \dfrac{\sum_{i \in S_{09}} w_{09} y_{09} / \sum_{i \in S_{09}} w_{09}}{\sum_{i \in S_{08}} w_{08} y_{08} / \sum_{i \in S_{08}} w_{08}} = AVG\_\hat{R}_{0809} \end{cases}$$

where $S_{08,} S_{09}$ are the samples in the year 2008 and 2009, $S_{08} \setminus C, and\ S_{09} \setminus C$ are the samples without certainties. Each $\hat{R}_1, \hat{R}_2$ has three versions. The final version is the one closest to 1.

The estimate of the variable of interest for state g and function code is:

$$\hat{y}_{gf} = \hat{R}_1 \hat{R}_2 y_{2007}$$

As we can see from the construction the step-wise ratios borrow the strength of previous year information.

III. Results

III.A 2009 Annual Survey of Public Employment and Payroll

The composite estimator was used to estimate the survey totals in each cell (state by function) of the ASPEP. As mentioned earlier, the composite estimator is the weighted average of the two estimators: the design-based and the synthetic. The composite balances out the instability of the unbiased due to small sample sizes with the synthetic quantity. The weight ϕ pulls the estimate to the design unbiased estimate when it has enough data, and towards the synthetic estimate when there is insufficient sample size in the small area (Rao, 2003).

By applying the methods described in Section 2, we created Table 3 which is a typical illustration of our data analysis. Table 3 is for the variable, Full-Time Equivalent Employment, in several randomly selected states. Those methods included a combination of Decision-based estimation and an application of a SAE method. The conclusions are as follows:

- When there were no observed sampled units, we used the synthetic estimate where the design-based direct estimates were not present. For example, there were no samples units in higher education in Arkansas or Oklahoma, so we obtained a reasonable synthetic estimate.
- The synthetic estimates were stable in small size areas where the design-unbiased estimates were very volatile.
- The modified direct estimates were closer to the census values.
- When the sample sizes were big enough, all the estimators performed well and they were close to each other.
- The composite using the modified direct estimator was close to the 2007 Census values most often.

Figure 3 shows the comparison among the composite estimate, synthetic estimate, design-based direct estimate (Horvitz-Thompson), and the 2007 data for the variable, Full Time Employees, in Alabama for all functions from the most recent Census of Governments. Figure 4 is a snapshot from Figure 3 which focuses on function codes 080, 081, 089, 091, and 092. Figure 4 shows the performance of the synthetic and the composite over the design-based estimate. Figures 3 & 4 show that when the sample sizes are relatively small the synthetic and the composite estimates outperformed the design-based estimates.

Note: Codes 080 and 091 are sewerage and water supplies which are problematic because respondents cannot separate the data for the two variables. Code 089 is problematic because it is a catch-all "All other" variable, which tends to be volatile.

Table 3: Comparison of Different Estimators in Various Sample Sizes

State	Function Code	y^S	y^D	y^{MD}	$y^{C_{HT}}$	$y^{C_{MD}}$	y^{EB}	y^{2007}	n
Alabama	Airport	430	497	457	464	444	497	424	14
Alaska	Airport	66	50	68	58	67	52	64	5
Arizona	Hospital	5018	2193	2433	3606	3726	2215	4767	2
California	Gas Supplies	263	289	276	276	267	294	265	3
Maryland	Electric Power	90	108	108	99	97	107	89	2
Arkansas	Higher Edu.	69	•	•	69	69	69	65	•
Oklahoma	Higher Edu.	118	•	•	118	118	118	116	•

Figure 3: Comparison of the Estimates from the Composite, Synthetic, and Horvitz-Thompson for the Variable Full-Time Employees in Alabama (all functions)

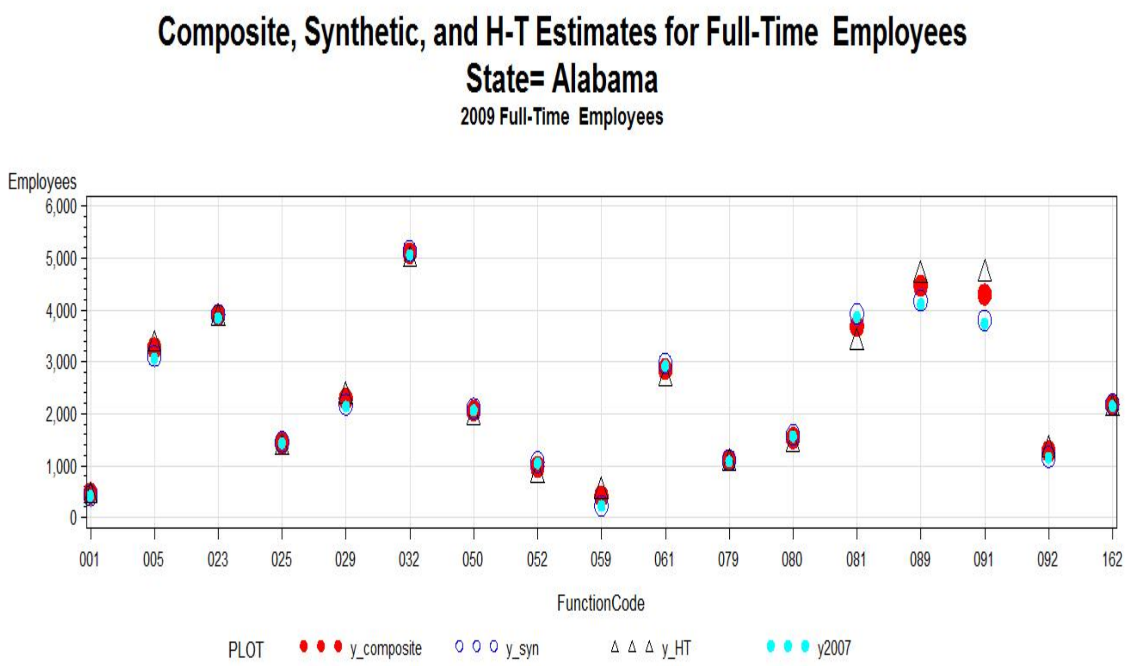
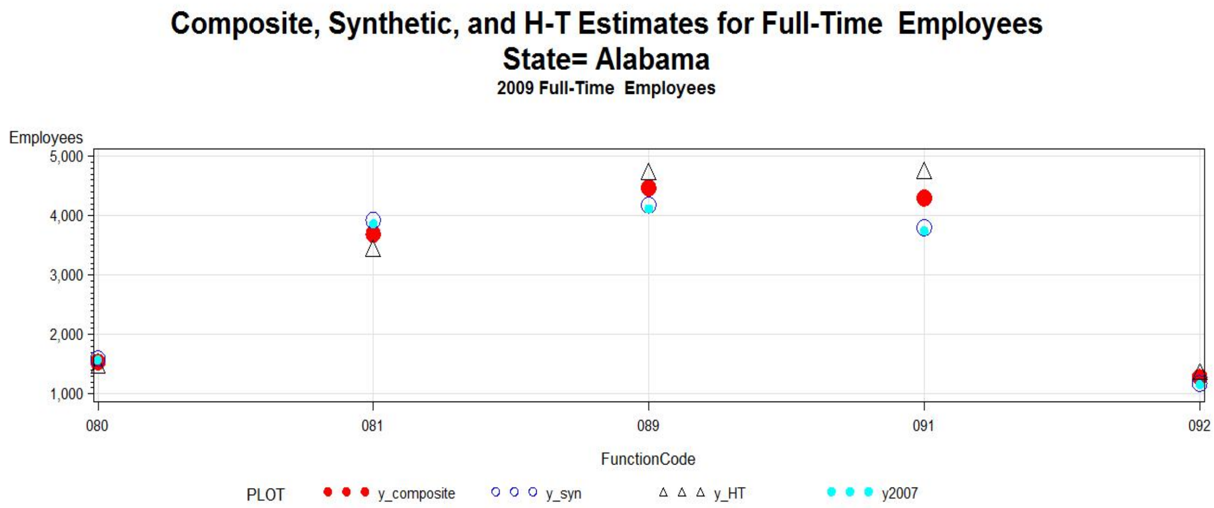


Figure 4: Comparison of the Estimates from the Composite, Synthetic, and Horvitz-Thompson for the Variable Full-Time Employees in Alabama



III.B Annual Finance Survey

Table 4 compared the estimates of four sectors for local governments at the national level for 2009, using the step-wise ratio method.

Table 4: Comparison of the Finance Estimates

AFS Category/Year	2007 Census	2008 Estimates, (CV)	2009 Estimates, (CV)
Revenue	1,467,480,331	1,558,796,314 (0.2%)	1,433,817,795 (0.3%)
Expenditure	1,468,690,544	1,547,210,282 (0.5%)	1,641,392,302 (0.3%)
Debt	1,005,383,331	1,036,405,779 (0.9%)	1,042,240,383 (0.4%)
Asset	877,766	2,390,948 (0.2%)	1,405,380 (0.5%)

We used New York 2009 census data to validate our estimation method. These data are available only for New York for a limited number of variables. The method yielded 89 percent of the cases where the step-wise ratio estimates are within 0.5 percent of the 2009 census. The cases out of 0.5 percent cut-off contained very volatile function codes like construction, or capital outlay. Those function codes represent the activities which are very difficult to model because the activities for those codes could be zero in one year but are active for the next year. If those activities are exclude in the estimation then the step-wise ratio estimates are within 0.5 percent of the 2009 New York census to 96.8 percent.

6. Conclusions

Bias of the synthetic estimator is the biggest disadvantage for synthetic estimation. Departures from the assumption may lead to large biases. Empirical studies have mixed results on the accuracy of synthetic estimators. The bias may not be estimated from the data.

The variance estimator for the complicated composite estimator derived from a Decision-based method needs separate research which will be presented in a future paper.

This paper presents two applications: Decision-based and Small Area Estimation methods. They were applied to the estimation of Annual Survey of Public Employment and Payroll. SAE provides the composite estimate which smoothes the design unbiased estimators in small areas by introducing the synthetic term. The synthetic estimate is more reliable when derived from the Decision-based estimates. This property cannot be obtained from a simple regression synthetic.

When these two methods are combined, we obtained better estimates than those of using direct estimators or with linear regression where the linear relationship is weak or even does not exist.

As for AFS estimation with validation from the New York census data, step-wise ratio is a good method to apply.

7. Future Research

We have some outstanding issues which need further research. We need to develop a simple and good variance estimator formula for the composite estimator other than a resampling method. Regarding the weight, $\hat{\phi}_g$, in the composite estimation method, we replaced $\hat{\phi}_g = 0.5$ when it was negative. Lahiri and Pramanik (2010) extended a method from Gonzalez & Waksberg (1973), which used Average Design-based Mean Squared Error (AMSE) to stabilized the $\hat{\phi}_g$. We will apply this method in our production in the future. We will also explore in more detail the application of the Empirical Bayes method with an alternative assumption other than normality. Finally, we will apply this method for other governments surveys, like the Annual Finance Survey (AFS).

As for AFS, we will apply the decision-based method to obtain reliable totals for each state, then use those totals and step-wise ratios to benchmark for each state and function code.

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